Journal of Computational and Applied Mathematics 11 (1984) 377–378
North-Holland

Letter Section

Convex cubic HERMITE-spline interpolation

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Received 4 July 1984

This letter is a corrigendum and addendum to the paper "Convex cubic HERMITE-spline interpolation", published in this journal, volume 9 (1983) 205–211.

The proof "(i) ⇒ (ii)" of Theorem 3.3 is incorrect. Therefore, we prove the equivalence of the three statements (i), (ii), (iii) by proving of "(i) ⇒ (iii) ⇒ (ii) ⇒ (i)" where the part "(iii) ⇒ (ii) ⇒ (i)" can be taken over from the original version. It remains to show the implication "(i) ⇒ (iii)". To do that we assume that (3.2) is solvable. Then we obtain at first

$$m_{i-1} \leqslant \tau_i, \quad i = 1, 2, \dots, n. \tag{$*$}$$

Hence, m_{n-1} is bounded by

$$m_{n-1} \leqslant \tau_n$$

and $\frac{1}{2}(3\tau_{n-1} - m_{n-2}) \leqslant m_{n-1} \leqslant 3\tau_{n-1} - 2m_{n-2}$. Once more using ($*$), (3.1) we have

$$\tfrac{1}{2}(3\tau_{n-2} - m_{n-3}) \leqslant m_{n-2} \leqslant 3\tau_{n-2} - 2m_{n-3},$$

$$a_1^{(n-2)} = 3\tau_{n-1} - 2\tau_n \leqslant m_{n-2} \leqslant \tau_{n-1}.$$

Finally, in the same manner we get the following bounds for m_{n-3}:

$$\tfrac{1}{2}(3\tau_{n-3} - m_{n-4}) \leqslant m_{n-3} \leqslant 3\tau_{n-3} - 2m_{n-4},$$

$$a_1^{(n-3)} = 3\tau_{n-2} - 2\tau_{n-1} \leqslant m_{n-3} \leqslant \tau_{n-2},$$

$$m_{n-3} \leqslant \tfrac{1}{2}(3\tau_{n-2} - 3\tau_{n-1} + 2\tau_n) = b_1^{(n-3)}.$$

Consequently, the validity of (a), (b) is shown for $i = n-2$ and $i = n-3$. Let us assume now that the inequalities

$$\begin{aligned} a_k^{(i)} \leqslant m_i \leqslant \tau_{i+1}, &\quad k = 1, 2, \dots, \left[\tfrac{1}{2}(n-i)\right], \\ m_i \leqslant b_k^{(i)}, &\quad k = 1, 2, \dots, \left[\tfrac{1}{2}(n-i-1)\right], \end{aligned} \tag{$**$}$$

are true for $i = n-2, n-3, \ldots, j+1$. From this and with the help of (3.1) we obtain

$$m_j \leqslant \tfrac{1}{2}\left(3\tau_{j+1} - a_k^{(j+1)}\right) = b_k^{(j)}, \quad k = 1, 2, \ldots, \left[\tfrac{1}{2}(n-j-1)\right],$$
$$m_j \geqslant 3\tau_{j+1} - 2b_k^{(j+1)} = a_{k+1}^{(j)}, \quad k = 1, 2, \ldots, \left[\tfrac{1}{2}(n-j-2)\right].$$
$$m_j \geqslant 3\tau_{j+1} - 2\tau_{j+2} = a_1^{(j)}.$$

Hence, $(**)$ is also valid for $i = j$. □

Remark. In addition to Theorem 3.3 we have: If the system (3.2) is solvable then all solutions can be obtained with Algorithm 1.

Proof. Let $m_0, \ldots, m_n$ be any solution of (3.2). Then we get from $(**)$

$$a_k^{(0)} \leqslant m_0 \leqslant \tau_1, \qquad k = 1, 2, \ldots, \left[\tfrac{1}{2}n\right],$$
$$m_0 \leqslant b_k^{(0)}, \qquad k = 1, 2, \ldots, \left[\tfrac{1}{2}(n-1)\right].$$

Hence, $m_0 \in [\underline{m}_0, \overline{m}_0]$. Now let $m_i \in [\underline{m}_i, \overline{m}_i]$ be true for $i = 0, \ldots, j-1$. Then we obtain from (3.1) and $(**)$

$$\tfrac{1}{2}\left(3\tau_j - m_{j-1}\right) \leqslant m_j \leqslant 3\tau_j - 2m_{j-1},$$
$$a_k^{(j)} \leqslant m_j \leqslant \tau_{j+1}, \qquad k = 1, 2, \ldots, \left[\tfrac{1}{2}(n-j)\right],$$
$$m_j \leqslant b_k^{(j)}, \qquad k = 1, 2, \ldots, \left[\tfrac{1}{2}(n-j-1)\right].$$

Therefore, $m_j \in [\underline{m}_j, \overline{m}_j]$ is also true. □

Acknowledgment

I am indebted to Prof. Dr. J.W. Schmidt who hinted me at the fault.